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Transient heat transfer to a fluid sphere suspended in an electric field

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Abstract—The unsteady heat transfer to a droplet suspended in an electric field is numerically investigated, for the case where the bulk of the resistance is in the droplet. It was found that, like the case of a translating droplet, there is a maximum steady-state Nusselt number. In this case, for a stationary drop in an electric field, the maximum steady Nusselt number was found to be about 30. An alternating direction implicit scheme was employed to integrate the energy equation. The range of interior Peclet numbers investigated, based on the maximum velocity in the droplet, was from 5 to 2000.

INTRODUCTION

When a uniform electric field is applied to a dielectric droplet suspended in a second dielectric medium, circulation inside the droplet is created. This internal flow field has been analytically evaluated by Taylor [1]. (See Fig. 1 for a schematic of the stream lines for this circulation inside the droplet created by a uniform electric field, as compared with the stream lines for a translating droplet, in the absence of an electric field.)

The heat transfer rates from a stationary droplet in an electrical field to the ambient fluid have been extensively studied for the special case where the preponderance of the resistance is in the continuous phase. Morrison [2] employed boundary-layer assumptions to estimate the Nusselt number for high exterior Peclet numbers. He found that the steady-state Nusselt number increases with the square root of the exterior Peclet number. Later Griffiths and Morrison [3] used a series truncation method to estimate the Nusselt number for lower exterior Peclet numbers ($0 \leqslant Pe \leqslant 60$). The Peclet number is based on the maximum velocity produced by the electric field. In the present study, as well as in the above investigations, the droplet remains suspended in the continuous medium.

Both of the above estimates for transfer rates only apply when the bulk of the resistance is in the continuous fluid (the external problem), and when the droplet has a much higher volumetric thermal capacity (ρC) than does the continuous phase. Traditional boundary-layer approximations are not valid in the interior region due to the recirculation in the interior region. If the bulk of the resistance is inside the droplet (the internal problem), a transient solution which accounts for this recirculation is required. (See Abramzon and Borde [4] for a good explanation of the

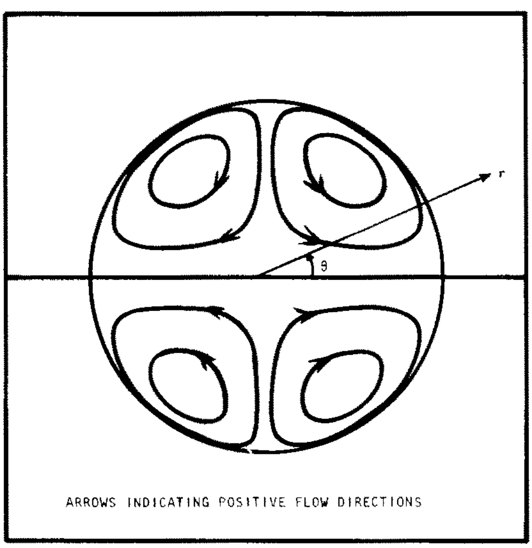

Fig. 1(a). Streamlines for a stationary drop in an electric field.

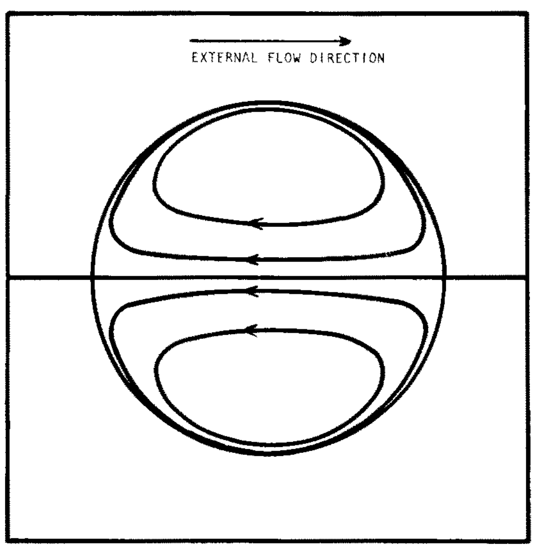

Fig. 1(b). Streamlines for a translating drop with no electric field.

NOMENCLATURE

- a radius of the droplet
- C specific heat
- d dielectric constant
- E electric field strength
- Fo Fourier number, $\alpha t/a^2$
- k thermal conductivity
- Nu Nusselt number based on diameter, see equation (9a)
- Pe Peclet number, $2Ua/\alpha$
- Q net rate of heat entering the drop
- R dimensionless radial coordinate, r/a
- r distance from droplet center
- T temperature
- t dimensional time
- U characteristic velocity of the droplet, see equation (5)
- u dimensionless radial velocity
- v dimensionless tangential velocity
- W ZR
- Z dimensionless temperature, $(T-T_s)/(T_{2,0}-T_s)$.

Greek symbols
- α thermal diffusivity
- θ tangential coordinate
- μ dynamic viscosity
- ρ density
- σ electrical resistivity.

Subscripts
- b bulk or average
- 0 initial condition of the droplet
- s surface or free stream
- 1 exterior, or based on exterior properties
- 2 interior, or based on interior properties.

difference between the external, internal and conjugate problems.)

FORMULATION

We will investigate the unsteady transport of heat to a droplet suspended in a uniform electrical field. The flow field is assumed to be fully developed. The droplet is suddenly exposed to a step change in the ambient temperature, with the bulk of the resistance being in the droplet.

The dimensionless energy equation for the droplet phase, assuming constant physical properties is

$$\frac{\partial Z}{\partial Fo_2}+\frac{Pe_2}{2}\left(u\frac{\partial Z}{\partial R}+\frac{V}{R}\frac{\partial Z}{\partial \theta}\right) = \frac{1}{R^2}\frac{\partial}{\partial R}\left(R^2\frac{\partial Z}{\partial R}\right)+\frac{1}{R^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial Z}{\partial\theta}\right) \quad (1)$$

$Z=(T_2-T_s)/(T_{2,0}-T_s)$, $Fo_2=\alpha_2 t/a^2$, $R=r/a$, and $Pe_2=2Ua/\alpha_2$ with U being the maximum velocity produced by the electrical field, a is the radius of the drop, the velocities are made dimensionless by U. The relation between Pe_2 (based on interior properties), and Pe_1 (based on exterior properties), is $Pe_2=Pe_1\alpha_1/\alpha_2$. Similarly $Fo_2=Fo_1\alpha_2/\alpha_1$, and $Nu_2=Nu_1k_1/k_2$.

The boundary conditions for equation (1) are then

At the interface ($R=1$)

$$Z=0 \text{ (i.e. the surface temperature is constant).} \quad (2)$$

Boundary condition (2) is a result of the preponderance of the resistance being in the droplet (i.e. the surface temperature is that of the free stream).

At the axis of symmetry ($\theta=0$, $\theta=\pi/2$)

$$\frac{\partial Z}{\partial\theta}=0. \quad (3)$$

The initial conditions are:

$$Z=1,\ \text{at } t=0,\ \text{everywhere.} \quad (4)$$

Taylor [1] gives the dimensional stream function values for the interior region as

$$\psi_2=Ua^2(R^3-R^5)\sin^2\theta\cos\theta. \quad (5)$$

with

$$U=\frac{-9E^2ad_2[(\sigma_1 d_1)/(\sigma_2 d_2)-1]}{8\pi(2+\sigma_1/\sigma_2)^2 5(\mu_1+\mu_2)}$$

The dimensionless velocities are then

$$u=\frac{1}{Ua^2R^2\sin\theta}\frac{\partial\psi}{\partial\theta}=(R-R^3)(2\cos^2\theta-\sin^2\theta), \quad (6)$$

$$v=\frac{-1}{Ua^2R\sin\theta}\frac{\partial\psi}{\partial R}=(5R^3-3R)\sin\theta\cos\theta. \quad (7)$$

The bulk dimensionless temperature, Z_b is defined as

$$Z_b=1.5\int_0^1\int_0^\pi ZR^2\sin\theta\,d\theta\,dR. \quad (8)$$

The Nusselt number is defined as

$$Nu_2=(2aQ)/4\pi a^2(T_s-T_b)k_2, \quad (9a)$$

which may be shown to be

$$Nu_2=-2/3\frac{d(\ln Z_b)}{dFo_2}. \quad (9b)$$

METHOD OF SOLUTION

Equation (1) was solved by a method similar to that used in ref. [4] for integration of the energy equation in the interior region of a translating sphere. The dimensionless temperature variable Z is transformed to

$$W = ZR. \tag{10}$$

Thus the transformed energy equation is

$$\frac{\partial W}{\partial Fo_2} + \frac{Pe_2}{2}\left[u\left(\frac{\partial W}{\partial R} - \frac{W}{R}\right) + \frac{v}{R}\frac{\partial W}{\partial \theta}\right] = \frac{\partial^2 W}{\partial R^2} + \frac{\cot\theta}{R^2}\frac{\partial W}{\partial \theta} + \frac{1}{R^2}\frac{\partial^2 W}{\partial \theta^2}. \tag{11}$$

The boundary conditions become

$$W(R=0,\theta,t) = 0, \tag{12}$$

$$W(R=1,\theta,t) = 0, \tag{13}$$

$$\frac{\partial W}{\partial \theta} = 0, \quad (\text{for } \theta = 0, \text{ and } \theta = \pi/2). \tag{14}$$

The initial conditions are

$$W(R,\theta,t=0) = R. \tag{15}$$

Equation (11) was integrated using an alternating direction implicit, ADI, scheme. A constant time step was employed for each run, with the time step ranging between $\Delta Fo_2 = 0.0007$ for lower Peclet numbers to $\Delta Fo_2 = 0.000025$ for $Pe_2 = 2000$.

A total of 61 nodes were used to approximate both the radial and tangential derivatives, that is $\Delta R = 1/60$, and $\Delta\theta = \pi/120$. Central differencing was employed for the convective derivatives, hence all spacial derivative approximations were second order accurate. One run was made with a 31×31 grid and a Peclet number of 2000, to check the precision of the spacial approximations. There was a 2% difference in the computed final Nusselt number and a 6% difference in the final computed bulk temperature, between the 31×31 and the 61×61 grid calculations.

For all calculations reported below a positive value of U was used, see equation (5). Several runs were made with a negative value of U (that is with a reversed flow direction). In all cases investigated ($20 \leqslant |Pe_2| \leqslant 2000$) no significant difference was found in either the Nusselt number or the bulk temperature with respect to flow direction.

RESULTS

Figure 2 shows the calculated Nusselt numbers as a function of Fourier number and Peclet number, (a) for Pe_2 from 5 to 200, and (b) for Pe_2 from 200 to 2000. At small times, conduction will be the dominant means of transfer to the droplet due to the sharp gradients near the interface. At larger times the convective component becomes significant for large values of Peclet numbers.

Initially the Nusselt number oscillates for the larger values of Peclet numbers. This oscillation is due to the internal circulation which supplies fresh fluid to the surface for rapid cooling. As time increases the Nusselt number approaches a steady value. This behavior is similar to that of a translating sphere (Abramzon and Borde [4]). For all the steady-state Nusselt numbers ($Fo \to \infty$) investigated, it is interesting to note that there appears to be a limiting maximum steady-state Nusselt number of about $Nu_2 = 30$ for electrically driven flows, this compares to the theoretical steady-state maximum limit from the analytical work of Kronig and Brink [5] that predicts a steady-state Nusselt number of 17.9 for translating spheres in the absence of an electrical field. It is reasonable that the maximum steady-state Nusselt number, for an electrically driven flow should be some what higher than the maximum steady-state Nusselt number for a translating drop. There are two circulation regions for the case of a stationary drop in an electric field, while the translating drop has only one circulation region. (See Fig. 1.)

One result of this maximum value of the steady-state Nusselt number is that there is a point of diminishing return for increases in heat transfer based upon the electrically induced velocities. This is not the case for the external problem if the drop has a volumetric

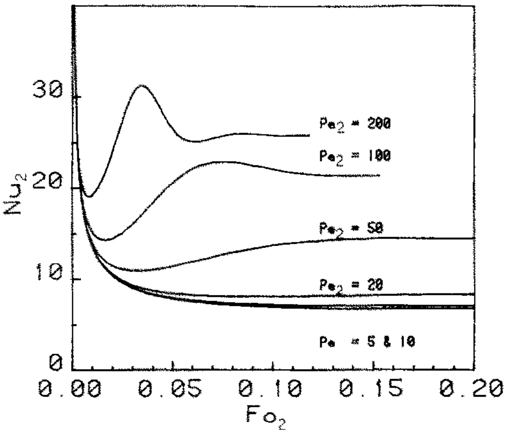

FIG. 2(a). Nusselt number: low to moderate Peclet numbers.

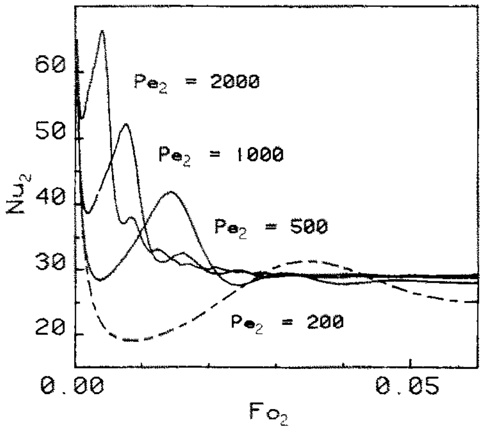

FIG. 2(b). Nusselt number: moderate to high Peclet number.

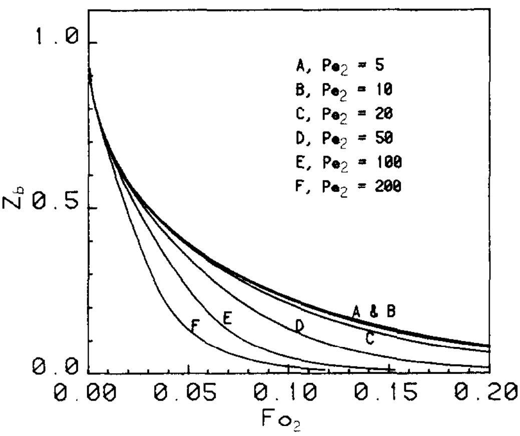

FIG. 3(a). Bulk temperature: low to moderate Peclet numbers.

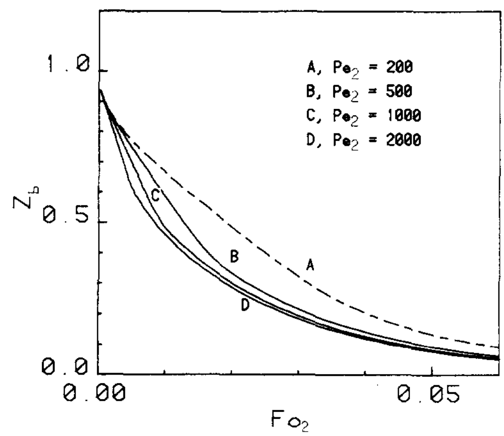

FIG. 3(b). Bulk temperature: moderate to high Peclet numbers.

thermal capacity (ρC) that is much greater than that of the continuous phase. Morrison [2] showed that for this case, with large Peclet numbers, the transfer rate increased with the square root of the Peclet number. The above results in heat transfer rates being proportional to the square root of the total power consumed. The Peclet number is proportional to E^2, with the total power also being proportional to E^2. Since the heat transfer rate (for this special case of the external problem) is proportional to the square root of the Peclet number, it follows that the heat transfer rate increases with the square root of the power consumed. Considering the internal problem, for Peclet numbers larger than about 500, the heat transfer rates become increasingly independent of the Peclet number (or power consumed).

This point of diminishing return is also evident in the plots of the bulk temperature profiles as a function of time for the various values of Peclet numbers. In Fig. 3 the bulk dimensionless temperatures are plotted as a function of Fourier number for the full range of Peclet numbers investigated.

The convergence to a maximum steady-state Nusselt number is shown in Fig. 4. At the lower Peclet numbers this steady-state Nusselt number is very near the theoretical value of $Nu_2 = 6.6$ for pure diffusion. As the Peclet number is increased the steady-state Nusselt number rapidly approaches a value of about 30 for large Peclet numbers.

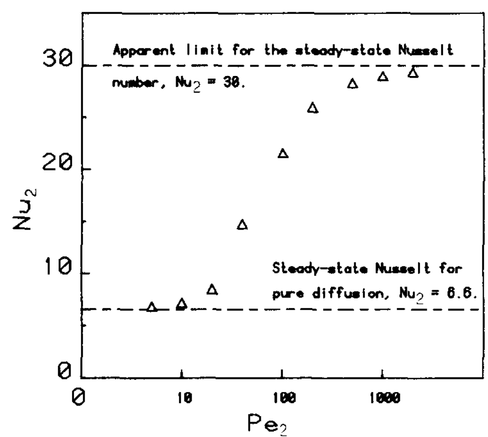

FIG. 4. Steady-state Nusselt number.

CONCLUSION

The transient energy equation has been integrated to obtain numerical predictions of the transfer rates for a stationary droplet in an electric field. There appears to be a maximum steady-state Nusselt number of about 30, which is independent of Peclet number, for high Peclet numbers. This value is about 67% higher than the maximum theoretical steady-state Nusselt number for a translating droplet. This asymptotic steady-state transfer rate would affect the operation of electrical extraction by limiting the economical operation ranges, for certain applications, to low and moderate Peclet numbers.

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TRANSFERT THERMIQUE VARIABLE POUR UNE SPHERE FLUIDE SUSPENDUE DANS UN CHAMP ELECTRIQUE

Résumé—On étudie numériquement le transfert thermique instationnaire pour une gouttelette suspendue dans un champ électrique, dans le cas où la résistance est principalement dans la gouttelette. On trouve que, comme pour une gouttelette en translation, il y a un nombre de Nusselt maximal d'état permanent. Dans ce cas, pour une goutte stationnaire dans un champ électrique, le nombre maximal de Nusselt est de l'ordre de 30. Un schéma implicite à direction alternée est employé pour intégrer l'équation d'énergie. Le domaine étudié de nombre de Péclet interne, basé sur la vitesse maximale dans la goutte, va de 5 à 2000.

INSTATIONÄRER WÄRMEÜBERGANG AN EINER IN EINEM ELEKTRISCHEN FELD SCHWEBENDEN FLUIDKUGEL

Zusammenfassung—Der instationäre Wärmeübergang an einem in einem elektrischen Feld schwebenden Tröpfchen wurde für den Fall, daß sich der Hauptwiderstand im Tröpfchen befindet, numerisch untersucht. Es wurde festgestellt, daß es—wie im Fall eines bewegten Tröpfchens—eine maximale stationäre Nusselt-Zahl gibt. Für einen stationären Tropfen in einem elektrischen Feld wurde eine maximale Nusselt-Zahl von etwa 30 ermittelt. Für die Integration der Energiegleichung wurde das implizite Verfahren der alternierenden Richtungen verwendet. Die Untersuchungen wurden für innere Peclet-Zahlen zwischen 5 und 2000 durchgeführt, basierend auf der maximalen Geschwindigkeit im Tröpfchen.

НЕСТАЦИОНАРНЫЙ ТЕПЛОПЕРЕНОС К ЖИДКОЙ СФЕРЕ В ЭЛЕКТРИЧЕСКОМ ПОЛЕ

Аннотация—Численно исследуется нестационарный теплообмен к капле, взвешенной в электрическом поле, когда сопротивлением обладает только капля. Обнаружено, что как и в случае изменяющейся капли стационарное число Нуссельта максимально. Для стационарной капли в электрическом поле максимальное стационарное число Нуссельта приблизительно равно 30. Уравнение энергии интегрируется с помощью неявной схемы переменного направления. Исследованный диапазон чисел Пекле, определенных по наибольшей скорости внутри капли, составил от 5 до 2000.